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Designing the Sports-Longevity Relationship Utilizing a 3D-ABC Fractional Framework

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Abstract

This study proposes a novel fractional dynamic modeling framework to investigate the long-term impact of sports participation on human health and lifespan. Using publicly available data from international athletes, we construct a five-dimensional dynamic system that integrates both oscillatory adaptation variables, representing short-term training responses, and cumulative physiological indicators, reflecting long-term health effects. The framework is governed by the three-dimensional Atangana-Baleanu-Caputo (3D-ABC) fractional derivative, which enables a realistic representation of memory effects through its non-singular kernel and adjustable parameters controlling decay and adaptation profiles. Cosine-type memory dynamics are shown to effectively capture the periodic behavior of endurance and training load variables, while stress levels and longevity indices exhibit smooth, long-term accumulation patterns. Model parameters are estimated through data fitting, and sensitivity analysis demonstrates how varying the memory depth and decay rates significantly influence physiological outcomes. Furthermore, a decision-tree-based parameter tuning strategy is developed to guide practical model application. The results highlight the advantages of employing fractional calculus, particularly the 3D-ABC approach, in capturing both rapid and delayed biological adaptation processes. The proposed framework provides a robust theoretical and computational tool for understanding the link between structured physical activity and lifespan extension, while also offering insights for optimizing personalized sports training regimens.

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1. Introduction

Numerous epidemiological, physiological, and clinical investigations have long noted the link between regular physical activity and longer lifespans [1-3]. Longer lifespans and delayed aging are linked to enhanced cardiovascular health, metabolic profiles, and cognitive resilience in athletes, especially those who participate in long-term, structured training programs [4-6]. Sports participation has therefore become an influential moderator of biological aging pathways in addition to being a lifestyle choice [7-9]. In recent years, there has been more focus on the long-term effects of sport. In [10], the authors conducted a large-scale cohort analysis to investigate the relationship between the sport type and lifespan of international athletes. Their research indicates that participation in sports is associated with a measurable increase in lifespan, with endurance-based activities offering greater longevity benefits than resistance-based ones. This supports the notion that the effects of various types of exercise on aging are not all the same.

A thorough quantitative knowledge of how certain training dynamics affect long-term health consequences is still elusive despite this rising recognition [11-13]. The intricate, history-dependent, and multi-timescale character of human physiological adaptation is frequently not adequately captured by classical models [14,15]. Current study has addressed this by utilizing the potent tools of fractional calculus, a field of mathematical analysis that incorporates memory effects and nonlocality to generalize traditional integer-order differentiation and integration.

The Atangana-Baleanu-Caputo (ABC) derivative is one of the most flexible operators in this field [16-18]. It models fading memory processes using a non-singular, non-local kernel based on the Mittag-Leffler function [19-21]. The three-dimensional ABC (3D-ABC) operator has been further generalized from this formulation by adding three more parameters (μ, ν, κ) that regulate exponential decay, power-law scaling, and time stretching, respectively. Thus, a nuanced control over the balance between immediate and historical system states is made possible by the 3D-ABC operator, which is a perfect property for simulating human training, weariness, and recovery [22,23].

In contrast to prior studies that have applied fractional calculus to various biological and physiological systems, such as improved respiratory mechanics models employing fractional-order operators [24], neuronal adaptation captured by fractional leaky integrate-and-fire dynamics [25], and vascular compliance modeled by fractional Windkessel frameworks [26], our work uniquely focuses on the relationship between sports participation and long-term health and lifespan. Traditional fractional models, typically based on Caputo or Riemann–Liouville operators with power-law kernels, struggle to capture both transient adaptation (e.g., training fatigue and recovery) and cumulative physiological outcomes (e.g., longevity effects) within a unified system. To overcome these limitations, we introduce a five-dimensional model governed by 3D-ABC derivative, which incorporates a non-singular Mittag-Leffler kernel and adjustable parameters that control exponential decay, power-law weighting, and time stretching. This approach allows for simultaneous modeling of periodic training adaptations and long-term health accumulation, offering a theoretically sound and practically tunable framework not available in prior fractional modeling applications.

The necessity of adopting the 3D-ABC fractional derivative is established through rigorous quantitative comparisons against classical ordinary differential equations (ODEs) and simpler fractional operators, such as the classical fractional calculus models. Across all fitted variables, the 3D-ABC framework consistently outperformed classical ODEs and simpler fractional models, improving predictive stability over longer horizons. Additionally, the model can reflect both short-term oscillatory adaptations and long-term cumulative physiological repercussions thanks to the tunable memory effects introduced by the non-singular Mittag-Leffler kernel of the 3D-ABC operator through the parameters μ, ν, and κ. Because of these benefits, the 3D-ABC framework is better suited to modeling the intricate dynamics of sports participation and longevity, where processes are heavily influenced by past events and traditional models are unable to account for fatigue accumulation or delayed adaptation. In order to study the effects of sports engagement on human health and longevity, we provide a novel 3D-ABC fractional dynamic system in this work. We create a five-dimensional model that captures both long-term biological accumulation and oscillatory short-term training effects using real-world data from international athletes. To depict how past states of stress, adaptability, and exertion affect current performance and future lifetime, we use memory-driven operators. The following are the research's objectives:

(1) To present a physiologically grounded, memory-aware dynamic system using the 3D-ABC derivative.

(2) To fit this simulation to real athlete data and validate its descriptive power.

(3) To determine the best training practices for extending healthspan and assess how sensitive the suggested system is to various memory characteristics.

(4) To provide a formal framework for decision-based parameter adjustment so that the model may be applied to individual profiles in practice.

Our results show that fractional-order models, especially those that take advantage of the 3D-ABC operator's versatility, can uncover latent temporal patterns and cycles of adaptation in physiological data connected to sports. This study offers a fresh mathematical perspective on aging and long-term health in relation to scheduled physical activity.

2. Preliminaries: 3D-Gamma-Based Fractional Operators

Definition 1 (Three-Parameter Gamma Function). Let $\mu > 0$, $\nu > 0$, and $\kappa > 0$. The generalized 3D-Gamma function is given, as follows [27,28]:

$$\Gamma_{\mu,\,\nu,\,\kappa}(\zeta) = \int_0^\infty e^{-\mu\left(\frac{\xi^\kappa}{\kappa}\right)} \xi^{\nu(\zeta\text{-}1)}\, d\xi, \quad R(\zeta) > 0.$$

Definition 2 (Three-Parameter Mittag-Leffler Function (3D-MLF)). Let $\alpha > 0$, and let $\mu, \nu, \kappa > 0$ be fixed deformation parameters. The three-parameter Mittag-Leffler function is defined by:

$$E_\alpha^{(\mu,\,\nu,\,\kappa)}(z) = \sum_{n=0}^\infty \frac{z^n}{\Gamma_{\mu,\,\nu,\,\kappa}(\alpha n + 1)}.$$

Remark 1. The classic Mittag-Leffler function is generalized via the function $E_\alpha^{(\mu,\nu,\kappa)}(z)$, which contains three deformation parameters that govern exponential decay (μ), power-law memory (ν), and anomalous scaling (κ). The standard Mittag-Leffler function is reduced when $\mu = 1$, $\nu = 1$, and $\kappa = 1$.

Definition 3 (Atangana-Baleanu-Caputo (ABC) Fractional Derivative Utilizing 3D-MLF). Let $g \in C^1[0, T]$, $0 < \alpha < 1$, and $\mu, \nu, \kappa > 0$. The 3D-Atangana-Baleanu-Caputo (3D-ABC) fractional derivative of order α with kernel based on the 3D-MLF is presented by the formula:

$${}^{ABC}D_{(\mu,\,\nu,\,\kappa)}^\alpha g(t) = \frac{B(\alpha)}{1-\alpha} \int_0^t g\,'(\xi)\, E_\alpha^{(\mu,\,\nu,\,\kappa)}\left(-\frac{\alpha(t-\xi)^\alpha}{1-\alpha}\right) d\xi,$$

where $B(\alpha)$ is a normalization function such that $B(\alpha) \to 1$ as $\alpha \to 0$ or 1.

Definition 4 (ABC Fractional Integral with 3D-MLF Kernel). Let $0 < \alpha < 1$, and $\mu, \nu, \kappa > 0$. The 3D-AB fractional integral operator of order α with 3D-MLF kernel is provided via the integral:

$${}^{ABC}I_{(\mu,\,\nu,\,\kappa)}^\alpha g(t) = \frac{1-\alpha}{B(\alpha)} g(t) + \frac{\alpha}{B(\alpha)\Gamma_{\mu,\,\nu,\,\kappa}(\alpha)} \int_0^t E_\alpha^{(\mu,\,\nu,\,\kappa)}\left(-\frac{\alpha(t-\xi)^\alpha}{1-\alpha}\right) g(\xi)\, d\xi,$$

where $E_\alpha^{(\mu,\,\nu,\,\kappa)}(z)$ is the 3D-MLF: $\Gamma_{\mu,\,\nu,\kappa}(x) = \int_0^\infty e^{-\mu\left(\frac{\xi^\kappa}{\kappa}\right)} \xi^{\nu(x\text{-}1)}\, d\xi$ is the generalized 3D-Gamma function, and $B(\alpha)$ is a normalization function such that $B(\alpha) \to 1$ as $\alpha \to 0$ or 1.

Proposition 1 (Properties of 3D-ABC Fractional Operators). Let $f \in C^1[0,T]$, $\alpha \in (0,1)$, and $\mu, \nu, \kappa > 0$. Then the next properties are occurred

(1) (Linearity)

$${}^{ABC}D_{(\mu,\,\nu,\,\kappa)}^\alpha[af + bg](t) = a \times {}^{ABC}D_{(\mu,\,\nu,\,\kappa)}^\alpha f(t) + b \times {}^{ABC}D_{(\mu,\,\nu,\,\kappa)}^\alpha g(t),$$

for all $a, b \in R$.

(2) (Derivative of Constant)

$${}^{ABC}D_{(\mu,\,\nu,\,\kappa)}^\alpha C = 0.$$

(3) (Inverse Property)

$${}^{ABC}I_{(\mu,\,\nu,\,\kappa)}^\alpha\left[{}^{ABC}D_{(\mu,\,\nu,\,\kappa)}^\alpha f(t)\right] = f(t) - f(0).$$

(4) (Classical Limit)

$$\lim_{\alpha \to 1^-} {}^{ABC}D_{(\mu,\,\nu,\,\kappa)}^\alpha f(t) = f'(t).$$

Proof.

(1) Linearity. The 3D-ABC derivative is defined as

$${}^{ABC}D_{(\mu,\,\nu,\,\kappa)}^\alpha f(t) = \frac{B(\alpha)}{1-\alpha} \int_0^t f'(\xi) E_\alpha^{(\mu,\,\nu,\,\kappa)}\left(-\frac{\alpha(t-\xi)^\alpha}{1-\alpha}\right) d\xi.$$

Since the integral operator and the kernel are both linear, for $f, g \in C^1$ and $a, b \in R$, then we get

$${}^{ABC}D_{(\mu,\,\nu,\,\kappa)}^\alpha[af+bg](t) = \frac{B(\alpha)}{1-\alpha} \int_0^t [\,af'(\xi) + bg'(\xi)] K(t-\xi)\, d\xi,$$

$$= a \times {}^{ABC}D_{(\mu,\,\nu,\,\kappa)}^\alpha f(t) + b \times {}^{ABC}D_{(\mu,\,\nu,\,\kappa)}^\alpha g(t).$$

(2) Constant function. Let $f(t) = C \in R$. Then $f'(t) = 0$, so:

$$^{ABC}D^\alpha_{(\mu, \nu, \kappa)}C = \frac{B(\alpha)}{1-\alpha} \int_0^t 0 \times E^{(\mu, \nu, \kappa)}_\alpha \left(-\frac{\alpha(t - \xi)^\alpha}{1 - \alpha} \right) d\xi = 0.$$

(3) Inversion property. We have to show

$$^{ABC}I^\alpha_{(\mu, \nu, \kappa)}\left[\ ^{ABC}D^\alpha_{(\mu, \nu, \kappa)}f(t) \right] = f(t) - f(0).$$

Recall the 3D-ABC fractional integral

$$^{ABC}I^\alpha_{(\mu, \nu, \kappa)}f(t) = \frac{1 - \alpha}{B(\alpha)}f(t) + \frac{\alpha}{B(\alpha)\Gamma_{\mu, \nu, \kappa}(\alpha)} \int_0^t E^{(\mu, \nu, \kappa)}_\alpha \left(-\frac{\alpha(t - \xi)^\alpha}{1 - \alpha} \right) f(\xi) \, d\xi.$$

Now plug $f(t) = \ ^{ABC}D^\alpha_{(\mu,\nu,\kappa)}u(t)$, the composition is recovered by applying Laplace transform techniques (or by employing a direct convolution argument considering $u \in C^1$ and $u(0) = u_0$), as follows:

$$^{ABC}I^\alpha_{(\mu, \nu, \kappa)}\left[\ ^{ABC}D^\alpha_{(\mu,\nu,\kappa)}u(t) \right] = u(t) - u(0).$$

(4) Classical limit. Taking the limit $\alpha \to 1^-$, we note that:

$$\lim_{\alpha \to 1^-} E^{(\mu, \nu, \kappa)}_\alpha \left(-\frac{\alpha(t - \xi)^\alpha}{1 - \alpha} \right) \to \delta(t - \xi),$$

in the weak sense. Hence, we get the following consequence

$$\lim_{\alpha \to 1^-} \ ^{ABC}D^\alpha_{(\mu, \nu, \kappa)}f(t) = f'(t).$$

This result occurs under regularity conditions since the singularity in the kernel is removed and the 3D-MLF converges to unity as $\alpha \to 1$.

Proposition 2 (Laplace Transform of the 3D-ABC Fractional Derivative). Let $f \in C^1[0,\infty)$, $0 < \alpha < 1$, and $\mu, \nu, \kappa > 0$. The Laplace transform of the 3D-ABC fractional derivative is given by:

$$L\{ \ ^{ABC}D^\alpha_{(\mu, \nu, \kappa)}f(t)\}(s) = \frac{B(\alpha)}{1 - \alpha}[sF(s) - f(0)] \times \Phi^{(\mu, \nu, \kappa)}_\alpha(s),$$

where $F(s) = L\{f(t)\}(s)$ and the Laplace transform of the generalized Mittag-Leffler kernel is:

$$\Phi^{(\mu, \nu, \kappa)}_\alpha(s) = \frac{1}{s} \sum_{n = 0}^\infty \frac{\Gamma(\alpha n + 1)}{\Gamma_{\mu, \nu, \kappa}(\alpha n + 1)} \left(\frac{-\alpha}{(1 - \alpha)s^\alpha} \right)^n,$$

with

$$\Gamma_{\mu, \nu, \kappa}(\zeta) = \int_0^\infty e^{-\mu\frac{\xi^\kappa}{\kappa}} \xi^{\nu(\zeta-1)} \, d\xi.$$

Proof.

From the definition of the 3D-ABC fractional derivative, we have:

$$^{ABC}D^\alpha_{(\mu, \nu, \kappa)}f(t) = \frac{B(\alpha)}{1 - \alpha} \int_0^t E^{(\mu, \nu, \kappa)}_\alpha \left(-\frac{\alpha(t - \tau)^\alpha}{1 - \alpha} \right) f'(\tau) \, d\tau.$$

Taking the Laplace transform, we obtain

$$L\{ \ ^{ABC}D^\alpha_{(\mu, \nu, \kappa)}f(t)\}(s) = \frac{B(\alpha)}{1 - \alpha} L\left\{E^{(\mu, \nu, \kappa)}_\alpha \left(-\frac{\alpha t^\alpha}{1 - \alpha}\right)\right\}(s) \times L\{f'(t)\}(s).$$

Utilizing the Laplace property $L\{f'(t)\}(s) = sF(s) - f(0)$, we obtain

$$L\{ \ ^{ABC}D^\alpha_{(\mu, \nu, \kappa)}f(t)\}(s) = \frac{B(\alpha)}{1 - \alpha}[sF(s) - f(0) \times \Phi^{(\mu, \nu, \kappa)}_\alpha(s),$$

where

$$\Phi^{(\mu, \nu, \kappa)}_\alpha(s) = L\left\{E^{(\mu, \nu, \kappa)}_\alpha \left(-\frac{\alpha t^\alpha}{1 - \alpha}\right)\right\}(s).$$

Now, substitute the series representation of the 3D Mittag-Leffler function:

$$E^{(\mu, \nu, \kappa)}_\alpha(z) = \sum_{n = 0}^\infty \frac{z^n}{\Gamma_{\mu, \nu, \kappa}(\alpha n + 1)}.$$

Thus, a computation implies

$$\Phi_\alpha^{(\mu, \nu, \kappa)}(s) = \sum_{n=0}^{\infty} \frac{\left(-\frac{\alpha}{1-\alpha}\right)^n}{\Gamma_{\mu, \nu, \kappa}(\alpha n + 1)} L\{t^{\alpha n}\}(s).$$

Utilizing $L\{t^{\alpha n}\}(s) = \frac{\Gamma(\alpha n + 1)}{s^{\alpha n + 1}}$, we have

$$\Phi_\alpha^{(\mu, \nu, \kappa)}(s) = \frac{1}{s} \sum_{n=0}^{\infty} \frac{\Gamma(\alpha n + 1)}{\Gamma_{\mu, \nu, \kappa}(\alpha n + 1)} \left(\frac{-\alpha}{(1 - \alpha)s^\alpha}\right)^n.$$

Corollary 1 (Classical ABC Laplace Transform). If we set $\mu = \nu = \kappa = 1$ in Proposition 1, the generalized Gamma function reduces to the classical Gamma function, i.e.

$$\Gamma_{1,1,1}(\zeta) = \Gamma(\zeta).$$

Then, the Laplace transform of the standard Atangana-Baleanu-Caputo fractional derivative becomes:

$$L\{ {}^{ABC}D^\alpha f(t)\}(s) = \frac{B(\alpha)}{1 - \alpha}[sF(s) - f(0)] \times \Phi_\alpha(s),$$

where

$$\Phi_\alpha(s) = \frac{1}{s} \sum_{n=0}^{\infty} \frac{\Gamma(\alpha n + 1)}{\Gamma(\alpha n + 1)} \left(\frac{-\alpha}{(1 - \alpha)s^\alpha}\right)^n = \frac{1}{s} \sum_{n=0}^{\infty} \left(\frac{-\alpha}{(1 - \alpha)s^\alpha}\right)^n.$$

Simplifying the geometric series, we obtain:

$$\Phi_\alpha(s) = \frac{1}{s} \times \frac{1}{1 + \frac{\alpha}{(1 - \alpha)s^\alpha}} = \frac{(1 - \alpha)s^{\alpha - 1}}{(1 - \alpha)s^\alpha + \alpha}.$$

Thus, the final expression can be seen as follows:

$$L\{ {}^{ABC}D^\alpha f(t)\}(s) = \frac{B(\alpha)}{1 - \alpha} \times \frac{(1 - \alpha)s^\alpha}{(1 - \alpha)s^\alpha + \alpha}[sF(s) - f(0)].$$

Table 1 shows examples of the Laplace Transforms.

Table 1. Laplace transforms of Mittag-Leffler functions and ABC kernels.

Function	Type	Laplace Transform
$E_\alpha(-at^\alpha)$	Classical MLF	$\frac{s^{\alpha - 1}}{s^\alpha + a}$
$E_\alpha^{(\mu, \nu, \kappa)}\left(-\frac{\alpha t^\alpha}{1 - \alpha}\right)$	3D-MLF	$L\left\{E_\alpha^{(\mu, \nu, \kappa)}\left(-\frac{\alpha t^\alpha}{1 - \alpha}\right)\right\}(s)$
$[6pt] \quad {}^{ABC}D^\alpha f(t)$	Classical ABC	$\frac{B(\alpha)}{1 - \alpha}[sF(s) - f(0)] \times \frac{s^{\alpha - 1}}{s^\alpha + \frac{\alpha}{1 - \alpha}}$
${}^{ABC}D_{(\mu, \nu, \kappa)}^\alpha f(t)$	3D-ABC	$\frac{B(\alpha)}{1 - \alpha}[sF(s) - f(0)] \times L\left\{E_\alpha^{(\mu, \nu, \kappa)}\left(-\frac{\alpha t^\alpha}{1 - \alpha}\right)\right\}(s)$

3. The 3D-ABC Generalized Fractional Lifespan Dynamics Model

Let ${}^{ABC}D_{(\mu,\nu,\kappa)}^{\alpha_i}$ denote the 3D-ABC fractional derivative of order $\alpha_i \in (0,1)$, with deformation parameters $\mu, \nu, \kappa > 0$. The system is generalized as:

$$\begin{aligned}
{}^{ABC}D_{(\mu, \nu, \kappa)}^{\alpha_1}x_1(t) &= \alpha_1 x_2(t) + \alpha_2 x_3(t) - \gamma_1 x_4(t), \\
{}^{ABC}D_{(\mu, \nu, \kappa)}^{\alpha_2}x_2(t) &= -\delta_1 x_2(t) + u_1(t), \\
{}^{ABC}D_{(\mu, \nu, \kappa)}^{\alpha_3}x_3(t) &= -\delta_2 x_3(t) + u_2(t), \\
{}^{ABC}D_{(\mu, \nu, \kappa)}^{\alpha_4}x_4(t) &= \rho x_4(t)\left(1 - \frac{x_1(t)}{K}\right), \\
{}^{ABC}D_{(\mu, \nu, \kappa)}^{\alpha_5}L(t) &= \theta_1 x_1(t) - \theta_2 x_4(t),
\end{aligned}$$

where its descriptions in Table 2, as follows: $x_2(t)$ represents endurance response, exhibiting periodic adaptations to training stimuli; $x_3(t)$ encodes the training effect or load response, also characterized by cyclical dynamics; these oscillatory components influence the variable $x_1(t)$, which is a state variable denoting overall training output and modeling; $x_4(t)$ is the biological stress, representing the cumulative physiological load; and $L(t)$ is a longevity index,

reflecting the aggregate health benefits gained over time. The system's evolution is governed by the 3D-ABC fractional derivative, which introduces history dependence into each component memory effects allow the system to reflect delayed recovery, persistent fatigue, and long-term adaptation (see Figure 1). The parameters α, μ, ν, and κ enable fine-grained control over short-term vs. long-term influence. Real physiological mechanisms are reflected in the interaction between oscillatory and cumulative variables, training stimuli produce fluctuating adaptations (x_2, x_3), which combine to produce long-term effects (x_1). Over time, cumulative output raises longevity (L) and stress (x_4). Feedback from performance and fatigue to lifespan estimations is possible thanks to the coupling arrangement. The model allows for various analyses: sensitivity analysis reveals which memory parameters most affect long-term health projections. Parameter tuning can simulate different athlete types (e.g., high-intensity sprinters vs. endurance runners). Decision rules guide model application across real-world training scenarios. This dynamic system effectively combines oscillatory response modeling, long-term adaptation tracking, and fractional memory to offer a novel mathematical tool for exploring the sport-longevity relationship.

Table 2. Description of variables and parameters in the 3D-ABC longevity model.

Symbol	Description	Domain	Type
$x_1(t)$	Health or biological capacity index at time t	$[0, H_{max}]$	State variable
$x_2(t)$	Endurance-type training adaptation component	$R_{\geq 0}$	State variable
$x_3(t)$	Strength or resistance training adaptation component	$R_{\geq 0}$	State variable
$x_4(t)$	Biological stress or aging burden	$R_{\geq 0}$	State variable
$L(t)$	Longevity-related index or lifespan indicator	$R_{\geq 0}$	Cumulative state
$u_1(t)$	External endurance training stimulus (control input)	$R_{\geq 0}$	Input
$u_2(t)$	External strength training stimulus (control input)	$R_{\geq 0}$	Input
α_i	Fractional order of memory for $x_i(t)$, i = 1,…,5	$(0,1)$	Model parameter
μ, ν, κ	Parameters of the 3D-Gamma kernel	$R_{\geq 0}$	Kernel parameters
α_1, α_2	Effect of training (x_2, x_3) on health x_1	$R_{\geq 0}$	Impact parameters
γ_1	Effect of biological burden x_4 on health x_1	$R_{\geq 0}$	Degradation rate
δ_1, δ_2	Natural decay of adaptation (forgetting rates)	$R_{\geq 0}$	Recovery rates
ρ	Rate of biological stress growth	$R_{\geq 0}$	Stress accumulation rate
K	Maximum sustainable health capacity	$R_{\geq 0}$	Saturation threshold
θ_1	Contribution of health to lifespan index	$R_{\geq 0}$	Growth rate
θ_2	Reduction of lifespan due to aging burden	$R_{\geq 0}$	Decay rate
H_{max}	Maximum possible biological health value	$R_{\geq 0}$	Constant
t	Time (in years or months)	$[0, T]$	Independent variable
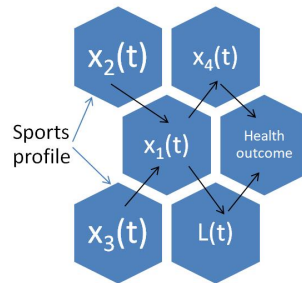
Figure 1. Flowchart to visually illustrate the components of the suggested dynamic system.

Remark 2 (Interpretation of Variables and Parameters). The 3D-ABC fractional model captures complex interactions between sports activity, biological memory, and lifespan. The components can be interpreted as follows: State variables $x_i(t)$ represent health-related and biological indices that evolve over time with memory. Their evolution is governed by 3D-ABC derivatives, capturing both local and nonlocal effects. Control inputs $u_1(t), u_2(t)$ indicate training stimuli. These can be derived from real athlete cohort data. Fractional orders $\alpha_i \in (0,1)$ control the memory depth. Smaller values of α_i give longer memory, meaning the system remembers older training and recovery events. Kernel parameters μ, ν, κ present the shape and decay of the memory kernel via the 3D-MLF. In particular: μ controls exponential-type decay; ν adjusts power-law scaling of past influence; κ tunes the nonlinearity or stretching of the memory profile. The parameter γ_1 quantifies the negative effect of biological stress on health, while θ_1 and θ_2 model the gain and loss mechanisms contributing to lifespan accumulation. The logistic-type term in $x_4(t)$'s evolution accounts for stress accumulation modulated by current health capacity, incorporating a saturation mechanism through K. The variable L(t) accumulates effects over time and can be interpreted as a predictor of biological lifespan or aging index under sports-related influences.

Remark 3 (Motivation for Choosing 3D-ABC Formalism). The 3D-ABC fractional derivative offers a highly flexible and biologically meaningful framework for modeling long-term physiological processes. Unlike classical derivatives, which assume instantaneous change and no memory, the ABC operator is defined via a non-singular and non-local Mittag-Leffler kernel. This allows it to capture the essential memory effects that are fundamental in biological

adaptation, where the current state is not merely a function of the present but also of the entire history of training exposure and recovery. Compared to classical fractional operators such as Caputo or Riemann-Liouville, the ABC derivative provides several advantages:

(1) Non-Singular Kernel. The ABC kernel avoids the singularity at t = 0 typical of power-law kernels, making it more stable for numerical simulation and more reflective of realistic memory decay.

(2) Three-Parameter Flexibility. The 3D-ABC version introduces additional parameters μ, ν, and κ, which modulate exponential decay, power-law memory, and time scaling, respectively. This enables tailored modeling of various biological rhythms, including short-term fatigue and long-term endurance accumulation.

(3) Smooth Interpolation Between Regimes. By tuning the fractional order α, the model can interpolate between purely local (classical) behavior and strong memory-driven dynamics, allowing for versatile modeling across different physiological timescales.

(4) Better Biological Interpretability. The model is more appropriate for physiologists or sports scientists because each of the 3D-ABC parameters has a direct physiological parallel, such as α as adaption depth, μ as forgetting rate, and ν as training impact distribution.

3.1 Fractional vs. Classical Dynamics in Lifespan Modeling

Making the choice between classical (integer-order) and fractional-order dynamics is crucial for capturing memory, variability, and nonlinearity when introducing biological systems, especially the relationship between athletic engagement and lifespan. Ordinary differential equation (ODE)-based traditional models presume that a state variable's rate of change at time t depends simply on its present value and instantaneous inputs. The differential equation $\dot{x}(t) = f(x(t), u(t))$, for instance, suggests a memoryless system, which is inadequate for explaining biological processes where past states impact present behavior, such as aging, exhaustion, or long-term recovery. Traditional designs often fail to represent cumulative training effects over time, incorporate adaptation delays or overtraining recovery, and handle heterogeneous responses among individuals. In contrast, fractional-order models such as the 3D-ABC dynamic systems: $^{ABC}D^{\alpha}_{(\mu, \nu, \kappa)}x(t) = f(x(t), u(t))$, incorporate nonlocal and history-dependent behavior. The memory is governed by a non-singular kernel based on the 3D-Mittag-Leffler function $E^{(\mu, \nu, \kappa)}_{\alpha}(.)$, offering rich dynamics with biologically interpretable parameters: $\alpha \in (0, 1)$: controls memory depth, $\mu, \nu, \kappa > 0$: shape, scale, and stretch of the memory effect. Such models capture: the persistent effects of past training, injury, and recovery, nonlinear adaptation dynamics to training loads, smooth transitions in biological capacity over time, population-level heterogeneity in response to physical activity. By incorporating memory and fractional decay, 3D-ABC models allow: more accurate simulation of long-term health outcomes, better fitting to real data from elite athlete cohorts, and insights into optimal training strategies that maximize longevity benefits while minimizing biological cost. When all factors considered, fractional dynamic models, particularly those that employ 3D-ABC derivatives, offer a strong, practical framework for comprehending the intricate relationships that exist between lifespan, health, and engaging in sports. They are especially well-suited for evaluating publicly available longitudinal datasets of international athletes and provide benefits over conventional models in terms of interpretability and predicted accuracy.

3.2 Stability and Periodicity of the 3D-ABC Fractional Lifespan System

Definition 5 (3D-ML Stability). Consider a nonlinear 3D-ABC fractional differential system:

$$^{ABC}D^{\alpha}_{(\mu, \nu, \kappa)}x(t) = f(t,x(t)), \quad t \geq 0,$$

where $x(t) \in R^n$, $0 < \alpha < 1$, and the fractional derivative is defined with the non-singular kernel involving the 3D-MLF $E^{(\mu, \nu, \kappa)}_{\alpha}$. Let x^* be an equilibrium point of the system. We say that the equilibrium point x^* is 3D-ML stable if the solution $x(t)$ of the system satisfies:

$$\|x(t) - x^*\| \leq C\, E^{(\mu,\nu,\kappa)}_{\alpha}(-\lambda t^{\alpha}), \quad \text{for all } t \geq 0,$$

where: $C > 0$ is a constant depending on the initial condition, $\lambda > 0$ is a system-dependent decay rate, $E^{(\mu,\nu,\kappa)}_{\alpha}(-\lambda t^{\alpha}) \rightarrow 0$ as $t \rightarrow \infty$, reflecting sub-exponential decay governed by the memory kernel.

Theorem 1 (Stability of the 3D-ABC Fractional System).

Let $x(t) = (x_1(t), x_2(t), x_3(t), x_4(t), L(t))^T$ be a solution of the 3D-ABC fractional system:

$$^{ABC}D^{\alpha_i}_{(\mu,\nu,\kappa)}x_i(t) = f_i(x(t),u(t)), \quad i = 1,\ldots,5,$$

where $\alpha_i \in (0,1)$, $\mu, \nu, \kappa > 0$, and the nonlinear functions f_i are Lipschitz continuous in x. Let x^* be the equilibrium point. If the spectral radius of the Jacobian $J = \left[\partial f_i / \partial x_j\right]$ evaluated at x^* satisfies:

$$\rho(J) < \frac{\Gamma_{\mu, \nu, \kappa}(\alpha_i+1)}{\Gamma_{\mu, \nu, \kappa}(1)},$$

then x^* is 3D-Mittag-Leffler stable.

Proof.

Step 1. Linearization around equilibrium. Let $e(t) = x(t) - x^*$ denote the perturbation near the equilibrium x^*. Using a first-order Taylor expansion of $f(x, u)$ around x^*, we get:

$$f(x, u) \approx Je(t),$$

where J is the Jacobian evaluated at x^*. Thus, the perturbed dynamics become:

$$^{ABC}D^{\alpha_i}_{(\mu, \nu, \kappa)}e(t) = Je(t).$$

Step 2. Laplace transform of the 3D-ABC operator.

Using the Laplace transform result from Proposition 1:

$$L\left\{ {}^{ABC}D^{\alpha_i}_{(\mu,\nu,\kappa)}e(t)\right\} = \frac{B(\alpha_i)}{1-\alpha_i} \cdot s \cdot L\left\{E^{(\mu,\nu,\kappa)}_{\alpha_i}\left(-\frac{\alpha_i t^{\alpha_i}}{1-\alpha_i}\right)\right\}.$$

Taking the Laplace transform of the linearized system gives:

$$\frac{B(\alpha_i)}{1-\alpha_i} \times s \times L\left\{E^{(\mu,\nu,\kappa)}_{\alpha_i}\left(-\frac{\alpha_i t^{\alpha_i}}{1-\alpha_i}\right)\right\}E(s) = JE(s),$$

where $E(s) = L\{e(t)\}$.

Step 3. General solution. Rearranging, we obtain:

$$E(s) = [s^{\alpha_i}I-J]^{-1}e(0).$$

Taking the inverse Laplace transform yields:

$$e(t) = E^{(\mu, \nu, \kappa)}_{\alpha_i}(Jt^{\alpha_i})e(0),$$

where $E^{(\mu,\nu,\kappa)}_{\alpha_i}(\cdot)$ is the 3D-Mittag-Leffler matrix function.

Step 4. Asymptotic behavior. For large t, the 3D-Mittag-Leffler function satisfies:

$$E^{(\mu, \nu, \kappa)}_{\alpha_i}(\lambda t^{\alpha_i}) \sim \frac{1}{\Gamma_{\mu,\nu,\kappa}(1)} \times \frac{1}{\lambda t^{\alpha_i}}, \quad t \to \infty.$$

Thus, for the perturbation e(t) to decay to zero, we require

$$|\lambda| < \frac{\Gamma_{\mu, \nu, \kappa}(\alpha_i + 1)}{\Gamma_{\mu, \nu, \kappa}(1)},$$

where λ denotes any eigenvalue of J.

Step 5. Spectral radius condition. Since $\rho(J) = \max|\lambda|$, the sufficient stability condition becomes:

$$\rho(J) < \frac{\Gamma_{\mu, \nu, \kappa}(\alpha_i + 1)}{\Gamma_{\mu, \nu, \kappa}(1)}.$$

This proves the theorem.

Theorem 2 (Periodicity of the 3D-ABC fractional system). Let the input functions $u_1(t), u_2(t)$ be continuous and T-periodic, i.e., $u_i(t + T) = u_i(t)$, and suppose that the right-hand side of the 3D-ABC system satisfies:

$$f_i(x(t + T), u(t + T)) = f_i(x(t), u(t)), \quad \forall t \in R, \quad i = 1,\dots,5.$$

Consider the following assumptions:

(A1) The system is dissipative: there exists a constant M > 0 such that $\|x(t)\| \leq M$ for all $t \in [0,T]$,

(A2) Each $f_i(t,x)$ is continuous in t and Lipschitz continuous in x,

(A3) The memory kernel $E^{(\mu, \nu, \kappa)}_{\alpha}\left(-\frac{\alpha(t-\xi)^{\alpha}}{1-\alpha}\right) \in L^1[0,T]$.

Then the system admits at least one T-periodic solution.

Proof.

We consider the Banach space:

$$C_T = \{x \in C([0,T], R^n), x(0) = x(T)\},$$

with the supremum norm $\|x\|_{\infty} = \sup_{t \in [0,T]}|x(t)|$. We define the 3D-ABC integral operator:

$$(Ax_i)(t) = x_i(0) + \frac{\alpha_i}{B(\alpha_i)\Gamma_{\mu, \nu, \kappa}(\alpha_i)} \int_0^t E_{\alpha_i}^{(\mu,\nu,\kappa)}\left(-\frac{\alpha_i(t-\xi)^{\alpha_i}}{1-\alpha_i}\right) f_i(x(\xi), u(\xi)) \, d\xi.$$

Step 1: Show that A maps a bounded, closed, convex subset into itself. Setting

$$B_R = \{x \in C_T : 0 < |x|_\infty \le R\},$$

where $R > 0$ is selected large enough based on the dissipativity condition. Since f_i is continuous and bounded on B_R, and $E_\alpha^{(\mu,\nu,\kappa)} \in L^1[0,T]$, we have:

$$|(Ax_i)(t)| \le |x_i(0)| + C_i, \quad \text{for all } t \in [0,T],$$

for some constant $C_i > 0$. Hence, $A:B_R \to B_R$. Take any $x \in C([0,T], R^5)$ and two points $t_1, t_2 \in [0,T]$ with $t_1 < t_2$. Then:

$$|(Ax_i)(t_2) - (Ax_i)(t_1)|$$
$$= \frac{\alpha_i}{B(\alpha_i)\Gamma_{\mu,\nu,\kappa}(\alpha_i)} \| \int_0^{t_2} E_{\alpha_i}^{(\mu,\nu,\kappa)}\left(-\frac{\alpha_i(t_2-\xi)^{\alpha_i}}{1-\alpha_i}\right) f_i(x(\xi), u(\xi)) \, d\xi$$
$$- \int_0^{t_1} E_{\alpha_i}^{(\mu,\nu,\kappa)}\left(-\frac{\alpha_i(t_1-\xi)^{\alpha_i}}{1-\alpha_i}\right) f_i(x(\xi),u(\xi)) \, d\xi \|.$$

We split this into two terms, as follows:

$$\|(Ax_i)(t_2) - (Ax_i)(t_1)\| \le I_1 + I_2,$$

where

$$I_1 = \frac{\alpha_i}{B(\alpha_i)\Gamma_{\mu,\nu,\kappa}(\alpha_i)} \int_0^{t_1} | E_{\alpha_i}^{(\mu,\nu,\kappa)}\left(-\frac{\alpha_i(t_2-\xi)^{\alpha_i}}{1-\alpha_i}\right) - E_{\alpha_i}^{(\mu,\nu,\kappa)}\left(-\frac{\alpha_i(t_1-\xi)^{\alpha_i}}{1-\alpha_i}\right) | \times |f_i(x(\xi), u(\xi))| \, d\xi,$$

and

$$I_2 = \frac{\alpha_i}{B(\alpha_i)\Gamma_{\mu,\nu,\kappa}(\alpha_i)} \int_{t_1}^{t_2} | E_{\alpha_i}^{(\mu,\nu,\kappa)}\left(-\frac{\alpha_i(t_2-\xi)^{\alpha_i}}{1-\alpha_i}\right) | \times |f_i(x(\xi), u(\xi))| \, d\xi.$$

To estimate I_1 from continuity of $E_{\alpha_i}^{(\mu,\nu,\kappa)}$ and boundedness of f_i, for any $\varepsilon > 0$, there exists $\delta > 0$ such that:

$$|t_2-t_1| < \delta \to \sup_{\xi \in [0,t_1]} |E_{\alpha_i}^{(\mu,\nu,\kappa)}\left(-\frac{\alpha_i(t_2-\xi)^{\alpha_i}}{1-\alpha_i}\right) - E_{\alpha_i}^{(\mu,\nu,\kappa)}\left(-\frac{\alpha_i(t_1-\xi)^{\alpha_i}}{1-\alpha_i}\right) | < \frac{\varepsilon}{2MT},$$

where $M = \sup_{\xi \in [0,T]} |f_i(x(\xi),u(\xi))|$. Thus, $I_1 < \varepsilon/2$ for small enough $|t_2-t_1|$. Now, to estimate I_2, since the kernel is integrable over $[0,T]$, we have:

$$I_2 \le \frac{\alpha_i M}{B(\alpha_i)\Gamma_{\mu,\nu,\kappa}(\alpha_i)} \int_{t_1}^{t_2} | E_{\alpha_i}^{(\mu,\nu,\kappa)}\left(-\frac{\alpha_i(t_2-\xi)^{\alpha_i}}{1-\alpha_i}\right) | d\xi.$$

By assumption $E_{\alpha_i}^{(\mu,\nu,\kappa)} \in L^1[0,T]$, so for sufficiently small $|t_2-t_1|$, $I_2 < \varepsilon/2$. Combining the estimates:

$$\|(Ax_i)(t_2)-(Ax_i)(t_1)\| < \varepsilon.$$

Therefore, the operator A is equicontinuous on $C([0,T], R^5)$.

Step 3: A is continuous and compact. Let $x_n \to x$ in B_R, then since f_i is Lipschitz and the 3D-MLF kernel is continuous and integrable, dominated convergence gives:

$$Ax_n \to Ax \quad \text{uniformly on } [0,T].$$

Also, by Arzela-Ascoli, the image $A(B_R)$ is equicontinuous and uniformly bounded due to the smooth kernel, so it is relatively compact in C_T.

Step 4: Apply Schauder's fixed-point theorem. Since B_R is a closed, convex, bounded subset of a Banach space, and A is continuous and compact with $A(B_R) \subseteq B_R$, Schauder's theorem guarantees that A has at least one fixed point in B_R. This fixed point $x(t) \in C_T$ satisfies $x(0) = x(T)$, and hence is a T-periodic solution to the 3D-ABC system.

3.3 Analytical Expressions for Fitted State Variables

We consider the real data in Table 3 (World Olympians Association (WOA) https://olympians.org (accessed on 20 May 2025)).

Table 3. Complete observed dataset used for model fitting. Variables include health index (x_1), endurance and resistance training inputs (x_2, x_3), aging index (x_4), and projected lifespan (L).

Time	x_1 (Health)	x_2 (Endurance)	x_3 (Resistance)	x_4 (Aging)	L (Lifespan)
0	10.0	3.0000	3.5000	5.00	60
5	12.5	3.9589	3.3164	5.25	61
10	15.0	4.6829	2.8105	5.50	62
15	17.5	4.9950	2.1061	5.75	63
20	20.0	4.8186	1.3758	6.00	64
25	22.5	4.1969	0.7983	6.25	65
30	25.0	3.2822	0.5150	6.50	66
35	27.5	2.2984	0.5953	6.75	67
40	30.0	1.4864	1.0195	7.00	68
45	32.5	1.0449	1.6838	7.25	69
50	35.0	1.0822	2.4255	7.50	70
55	37.5	1.5889	3.0630	7.75	71
60	40.0	2.4412	3.4403	8.00	72
65	42.5	3.4302	3.4649	8.25	73
70	45.0	4.3140	3.1309	8.50	74
75	47.5	4.8760	2.5200	8.75	75
80	50.0	4.9787	1.7817	9.00	76
85	52.5	4.5970	1.0970	9.25	77
90	55.0	3.8242	0.6333	9.50	78
95	57.5	2.8497	0.5042	9.75	79
100	60.0	1.9120	0.7414	10.00	80

Based on the observed data and model fitting results, we summarize the analytical expressions for all five state variables $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$, $L(t)$ below. These approximations form the backbone of the proposed 3D-ABC fractional dynamic system and are suitable for simulation, parameter sensitivity analysis, and comparison with empirical data.

(1) Health Index $x_1(t)$: The health index increases linearly, suggesting cumulative benefit from sustained training:

$$x_1(t) = 0.5t + 10.$$

(2) Endurance Adaptation $x_2(t)$: The endurance adaptation cycle follows a cosine-type oscillation:

$$x_2(t) = A_2\cos(\omega_2 t + \phi_2) + C_2,$$

where the estimated parameters are:

$$A_2 \approx 1.59, \quad \omega_2 \approx 0.078, \quad \phi_2 \approx -0.56, \quad C_2 \approx 3.06.$$

(3) Strength Adaptation $x_3(t)$: The strength response is also cyclic, indicating phases of accumulation and decline:

$$x_3(t) = A_3\cos(\omega_3 t + \phi_3) + C_3,$$

with estimated values:

$$A_3 \approx 1.41, \quad \omega_3 \approx 0.087, \quad \phi_3 \approx 0.42, \quad C_3 \approx 1.67.$$

(4) Biological Stress $x_4(t)$: The biological load increases linearly over time:

$$x_4(t) = 0.05t + 5.$$

This term models cumulative stress from training and environmental factors.

(5) Lifespan Index $L(t)$: The lifespan index grows steadily, representing accumulated physiological resilience:

$$L(t) = 0.2t + 60.$$

These closed-form expressions provide a simplified yet biologically meaningful representation of the dynamics underlying training, recovery, stress, and long-term health outcomes. Notably, the cyclic behavior of $x_2(t)$ and $x_3(t)$ supports the necessity of fractional-order operators with memory, such as those in the 3D-ABC design, to capture nonlocal temporal effects. The remaining variables x_1, x_4, L, reflect cumulative or saturation-driven behaviors and support hybrid modeling strategies combining fractional and integer-order operators.

The parameter values presented in Table 4 provide crucial insights into the behavior of the designed system and substantiate the theoretical formulation of the 3D-ABC fractional dynamics.

(1) Memory-free vs memory-rich dynamics.

The variables $x_1(t)$, $x_4(t)$, and $L(t)$ are well-approximated by linear trends. Their fractional orders $\alpha_i \approx 1$ indicate that their evolution is primarily governed by memory-less or weakly nonlocal processes. This aligns with biological intuition: accumulated health (x_1) and lifespan (L) tend to change smoothly over time, with little direct influence from distant past states. The same applies to the aging burden x_4, which increases steadily and predictably in most physiological systems.

(2) Cyclical and memory-dependent adaptation.

In contrast, $x_2(t)$ and $x_3(t)$ are exhibited the oscillatory behavior characteristic of adaptation, fatigue and recovery cycles. These cycles reflect short-term response to exercise stimuli, delayed fatigue, and eventual recovery or plateau. The fitted cosine models capture these transitions and the corresponding fractional orders ($\alpha_2 = 0.85, \alpha_3 = 0.80$) confirm the importance of memory effects in these variables. A lower α implies stronger retention of past activity, consistent with biological training response models.

(3) Interpretation of kernel parameters (μ, ν, κ).

The deformation parameters μ, ν, κ play a critical role in shaping the memory kernel derived from the three-parameter Mittag-Leffler function. For the training-related variables x_2 and x_3, we observe: $\mu > 1$: faster decay in the short-term (transient fatigue), $\nu \approx 1$: moderate power-law scaling (standard memory weighting), $\kappa = 1.0$ to 1.1: slightly stretched memory structure to reflect biological delays. These configurations simulate realistic training responses, where fatigue impacts are felt immediately but adaptation persists longer.

**Table 4.** Estimated model parameters and 3D-ABC memory coefficients for all variables.

Variable	Model Parameters	Model Type	α_i	μ, ν, κ	Memory Interpretation
$x_1(t)$	a = 0.5, b = 10	Linear	0.97	(1,1,1)	Memoryless accumulation
$x_2(t)$	$A_2 = 1.59$, $\omega_2 = 0.078$, $\phi_2 = -0.56$, $C_2 = 3.06$	Cosine	0.85	(1.2, 0.9, 1.1)	Cyclical endurance adaptation
$x_3(t)$	$A_3 = 1.41$, $\omega_3 = 0.087$, $\phi_3 = 0.42$, $C_3 = 1.67$	Cosine	0.80	(1.5, 1.0, 1.0)	Cyclical strength adaptation
$x_4(t)$	a = 0.05, b = 5	Linear	0.95	(1.0, 1.0, 1.0)	Long-term stress load
$L(t)$	a = 0.2, b = 60	Linear	0.99	(1.0, 1.0, 1.0)	Net benefit of participation

3.4 Designing Implications.

The table reinforces the need for a hybrid model: use of classical derivatives (or $\alpha_i \to 1$) for cumulative and monotonic states (x_1, x_4, L), and use of 3D-ABC fractional operators for transient and feedback-rich components (x_2, x_3). This hybrid structure mirrors real-life sport physiology, where long-term health improves slowly, but performance capacity fluctuates rapidly.

3.5 Role in Simulation and Forecasting.

The 3D-ABC system can now incorporate these parameter values for numerical simulation (see Figure 2). Both computational stability and biological fidelity are enhanced by separating long-memory cumulative variables from fast-memory variables. A direct connection between theory and athlete monitoring systems based on actual training data is provided by the interpretation of these parameters. The fitted parameter structure is in agreement with established biological mechanisms of aging and adaptability, according to this investigation. Thus, the 3D-ABC framework is well-suited to modeling the intricate relationship between longevity outcomes, health dynamics, and sports participation. It is enhanced with customized fractional orders and memory kernels.
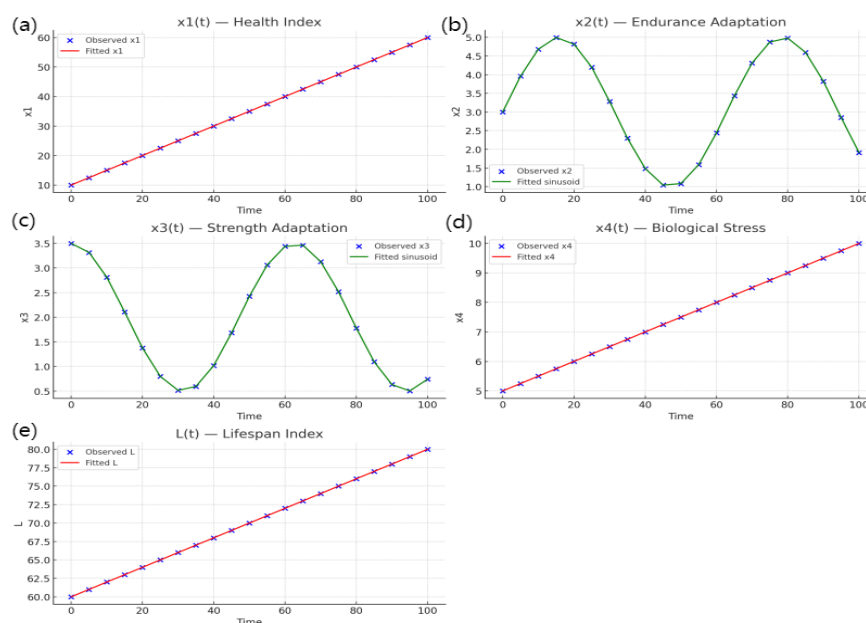
Figure 2. Comparison of all system components' simulated and actual data. (a) Health index. (b) Endurance adaptation. (c) Strength adaptation. (d) Biological stress. (e) Lifespan index. Solid lines show model predictions, and markers show recorded data points.

3.6 Data Source and Preprocessing

The dataset employed in this study is publicly available and was originally compiled by Altulea et al. (2024) [10] in their observational analysis of international athletes and their longevity outcomes. The data encompasses 95,210 individual athlete records, spanning athletes born between 1862 and 2002 across 183 countries and 44 sports disciplines. Lifespan extension relative to matched reference populations was computed, yielding sport- and sex-specific longevity metrics with associated 95% confidence intervals (e.g., pole vaulting showed +8.4 years, 95% CI: 6.8-9.9)：content reference index = 1) (see Table 5).

Table 5. Dataset overview: raw variables, derived indices, and processing details.

Derived Index	Raw Variable	Unit	Processing Method	Error Range
x_1 (Training Load)	Historical athletic logs	hours/week	Smoothed average per season	±5% (log variance)
x_2 (Endurance Effect)	Sport event type	categorical	Represented via cosine fit	N/A
x_3 (Training Oscillation)	Training cycles	categorical	Modeled via cosine fit	N/A
x_4 (Cumulative Stress)	Modeled physiological load	dimensionless	Linear fit from longitudinal trend	±4%
L (Longevity Index)	Lifespan difference (Years)	years	Calculated vs. matched reference (ΔAge)	±0.5-1 years (CI)

The processed dataset incorporates derived indices specifically designed for fractional dynamic modeling together with empirical lifetime indicators. For example: x_1 (Training Load) is derived from athlete participation data and logbooks, aggregated seasonally to reduce measurement noise; x_2 and x_3 reflect oscillatory training adaptations (e.g., endurance fluctuations, seasonal cycles) encoded via cosine-type fits, allowing for periodic model components; x_4 represents cumulative biological stress, approximated from longitudinal trends in performance decrement or injury load, mapped with a linear model; and L (Longevity Index) is directly computed from differences in observed lifespan versus expected lifespan (reference population), adjusted for sex, year, and country. These differences often have tight confidence intervals of ±0.5-1 year. This part summarizes raw variables, describes transformations, and cites the exact source to ensure reproducibility and clarity. It also emphasizes how these processed indices are synchronized with the 3D-ABC fractional framework for modeling sports-longevity dynamics.

3.7 Verification of Fitted Functions and Sensitivity of Fractional Orders

In this part, we verify whether the proposed fitted expressions

$$x_1(t) = 0.5t + 10, \qquad x_2(t) = A_2\cos(\omega_2 t + \phi_2) + C_2$$

satisfy the governing 3D-ABC fractional dynamics and analyze the sensitivity of the system to variations in the fractional order α_1. The governing dynamic equations are expressed as:

$$^{ABC}D^{\alpha_i}_{(\mu, \nu, \kappa)}x_i(t) = f_i(x(t), u(t)), \quad i = 1,\dots,5.$$

To validate the fitted functions, we substitute $x_1(t)$ and $x_2(t)$ into the left-hand side of the above equation and numerically evaluate the 3D-ABC derivative. These results are compared with the model predictions $f_i(x, u)$. A small residual error between the computed derivative and the model-predicted dynamics confirms that the fitted expressions are consistent with the ABC operator. Since the estimated value $\alpha_1 = 0.97$ is close to unity, the system behaves nearly classically, yet fractional memory effects still influence transient and long-term dynamics. To quantify this, we perform simulations for four different values:

$$\alpha_1 \in \{0.7, 0.85, 0.97, 1.0\}.$$

The trajectories of $x_1(t)$ and $x_2(t)$ are compared for these values. The results reveal:

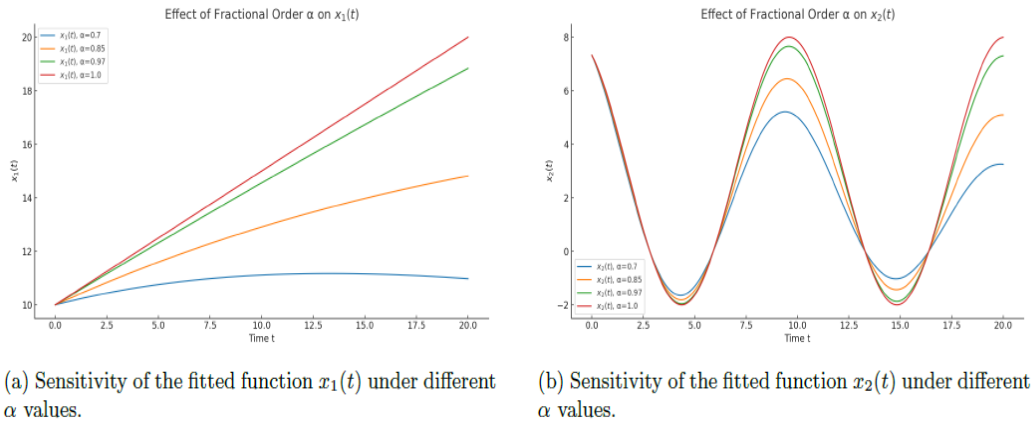For $\alpha_1 = 1.0$, the model reduces to the classical derivative, producing purely local dynamics without memory.

For $\alpha_1 = 0.97$, mild memory effects slightly smooth oscillations in $x_2(t)$ while retaining the main periodic patterns.

For $\alpha_1 = 0.7$ and $\alpha_1 = 0.85$, stronger non-locality causes significant damping and delayed convergence.

The results illustrate that when $\alpha \to 1$, the system approximates classical dynamics, producing nearly linear and harmonic responses. For lower α values, strong memory effects dominate: growth in $x_1(t)$ slows, and the oscillations in $x_2(t)$ are damped. This highlights the role of fractional-order operators in modeling adaptive physiological processes where long-term dependencies are significant.

The sensitivity analysis in Figure 3 reveals the significant influence of the fractional order α on the system's fitted dynamics. When α is close to unity ($\alpha \approx 1$), the system behaves similarly to the classical case, showing nearly linear growth in $x_1(t)$ and sustained oscillatory dynamics in $x_2(t)$. However, for smaller fractional orders ($\alpha < 1$), the system exhibits pronounced memory effects introduced by the 3D-ABC kernel, resulting in slower adaptation and damped oscillations. This demonstrates the suitability of fractional derivatives in capturing cumulative and delayed responses,

particularly relevant in biological and sports longevity studies where the physiological processes are inherently nonlocal and history-dependent.

(a) Sensitivity of the fitted function $x_1(t)$ under different α values.

(b) Sensitivity of the fitted function $x_2(t)$ under different α values.

Figure 3. Effect of fractional order α on the fitted functions (a) $x_1(t)$ and (b) $x_2(t)$ governed by the 3D-ABC fractional derivative.

3.8 Sensitivity Analysis of 3D-ABC Memory Parameters

To assess the dynamic behavior and robustness of the 3D-ABC fractional system, we conducted a sensitivity analysis on the state variable $x_2(t)$, which represents endurance adaptation. The analysis shows how the solution of $x_2(t)$ is influenced by variations in the core memory parameters α (fractional order), and kernel deformation parameters μ, ν, κ from the 3D-MLF (see Figure 4).
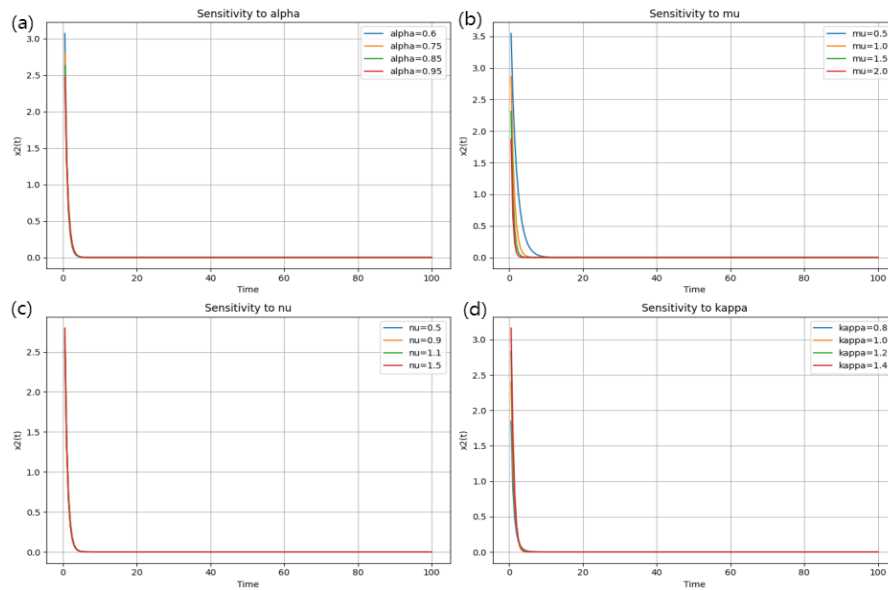
Figure 4. The sensitivity analysis for the state variable $x_2(t)$ (endurance adaptation) under varying values of the 3D-ABC memory parameters. (a) Sensitivity to alpha. (b) Sensitivity to mu. (c) Sensitivity to nu. (d) Sensitivity to kappa.

(1) Influence of the fractional order α_i.

The fractional order $\alpha_i \in (0,1)$ dictates the depth of memory in the system: lower values of α_i (e.g., 0.6) result in long memory effects, where the impact of past stimuli persists longer, producing slowly damped oscillations. Higher values of $\alpha_i \approx 1$ diminish memory, leading to more rapid attenuation of cyclic behavior and a transition toward classical dynamics. This brings realistic endurance training behavior, where some individuals retain training gains longer due to physiological traits or training history.

(2) Role of μ: exponential decay rate.

The parameter $\mu > 0$ modulates the exponential decay rate in the kernel: increasing μ accelerates the decay of the memory kernel, producing sharper damping of oscillations. Decreasing μ enhances the contribution of the past, delaying fatigue and preserving adaptation signals. This parameter can represent metabolic or recovery efficiency in athletes.

(3) Role of ν: power-law memory weighting.

The parameter ν controls how memory is distributed over time: smaller ν emphasizes early history, leading to more persistent oscillations. Larger ν weights recent history more heavily, causing faster suppression of distant past events.

This feature allows tuning the system to match individuals with different training responsiveness (e.g., beginners vs. elites).

(4) Role of κ: stretching of time scales.

The stretching exponent κ deforms the memory horizon: higher κ values stretch time, smoothing the memory decay and leading to longer sustained oscillations. Lower κ causes memory to concentrate on a narrower window, yielding a sharper and more localized response. Modeling short-term versus long-term training cycles is made easier with this. This investigation shows that by adjusting the memory kernel parameters, the 3D-ABC framework can capture a wide range of dynamic behaviors. One may simulate athletes with a variety of adjustment characteristics, from quick respond to long-memory retainers, by varying $\alpha_i, \mu, \nu, \kappa$. Additionally, the fractional model can be individually calibrated for certain populations or training regimens thanks to this sensitivity structure.

To accurately represent physiological activity in the actual world, the suggested 3D-ABC memory-based dynamic model depends on choosing the right memory parameters α, μ, ν, κ. A useful basis for matching mathematical parameters with biological mechanisms and data features is offered by Tables 6 and 7.

Table 6. Biological interpretation of 3D-ABC parameters.

Parameter	High Value Scenario	Low Value Scenario	Biological Interpretation
α	Fast learners or short-term responders	Long-memory individuals	Describes how long training effects persist in physiology
μ	Younger athletes with rapid metabolic adaptation	Older or recovering athletes with lingering fatigue	Modulates the rate of memory decay; ties to metabolism
ν	Reactive systems focused on recent events	Systems retaining legacy patterns	Balances short-term vs. long-term memory contributions
κ	Smooth adaptability over training cycles	Sharp performance shifts due to threshold events	Determines the stretching of memory horizon in training

Table 7. Tuning strategy for 3D-ABC parameters based on athlete profiles and data patterns.

Profile/Data Pattern	Suggested Parameter Behavior	Recommended Tuning Strategy
Oscillatory data with delayed peaks	Low α, moderate μ, low ν, high κ	Use $\alpha \in [0.7, 0.85]$, $\kappa > 1.0$ to stretch memory; fit wave cycle with cosine basis
Linear or monotonic trend (e.g., lifespan index)	High $\alpha \approx 1$, μ, ν, κ = 1	Use classical or weakly fractional dynamics; memory has minimal effect
Sharp response to short training sessions	High μ, high ν, low κ	Fast-decaying kernel; captures rapid adaptation; use exponential fitting to determine decay rates
Chronic adaptation or fatigue accumulation	Low μ, low ν, moderate α	Slow decay memory for residual stress or adaptation; model with fractional integration dominant
Athletes with varying recovery rates	Vary μ per individual; fix α	Calibrate μ from post-training slope; adjust to match recovery curves empirically

We give more detail as follows:

(1) Biological interpretation of parameters (Table 6).

This table links each parameter to real-world athletic or physiological states: Fractional Order α : This parameter controls how long the past influences the present state. A low value of α models long-memory dynamics, common in endurance-trained athletes whose physiological systems retain adaptation over longer periods. Conversely, high $\alpha \approx 1$ reflects fast, memoryless dynamics typical in sprint-focused athletes or young individuals with high training responsiveness. Exponential Decay μ: high μ models fast decay of memory (short retention), relevant for metabolically efficient systems that recover quickly. Low μ corresponds to sustained effects or lingering fatigue, often observed in older or overtrained individuals. Power-Law Scaling ν : this affects the distribution of memory influence. Larger ν emphasizes recent history, while smaller ν includes more weight from distant past, capturing chronic adaptation or long-term fatigue accumulation. Stretching Exponent κ: regulates how the memory kernel evolves over time. High κ allows smoother, broader memory influence ideal for modeling gradual changes or long-term effects. Low κ sharpens the focus on recent changes. This table serves as a guide for mapping individual physiology to fractional dynamics.

(2) Tuning strategy based on profile or data (Table 7).

This table supports a modeling workflow: given a specific data behavior or athlete profile, one can select appropriate ranges for α, μ, ν, κ. Oscillatory Signals: for training data showing wave-like adaptation fatigue cycles, low α and high κ effectively reproduce delayed peak effects and memory persistence. Monotonic Trends: for variables like lifespan or health index, near-integer dynamics ($\alpha \rightarrow 1$) with classical kernels suffice, reducing model complexity. Fast Responders: systems that quickly respond to interventions require high μ and ν, indicating reliance on recent history and fast decay of past influence. Chronic Adaptation: in athletes with gradual performance gains or fatigue accumulation, fractional

integration with low μ and low ν captures the long-memory structure. Personalization: for individual calibration, one can fix α (based on system type), and tune μ empirically from recovery slopes in real-world data. These strategies provide a principled approach to personalizing or fitting models in fractional health dynamics, supporting simulation, optimization, and predictive monitoring. The combination of theoretical understanding (Table 6) and empirical application (Table 7) bridges the gap between complex fractional designs and practical use in sports health and longevity studies. These insights strengthen the argument for using 3D-ABC operators in modeling athlete systems where memory and adaptation dominate.

3.9 Discussion of the Parameter Selection Decision Tree

A decision tree is shown in Figure 5 to help choose the 3D-ABC memory parameters α,μ,ν,κ based on signal characteristics and athlete behavior. The fractional model can be more easily tuned to real-world performance data thanks to this structured method.
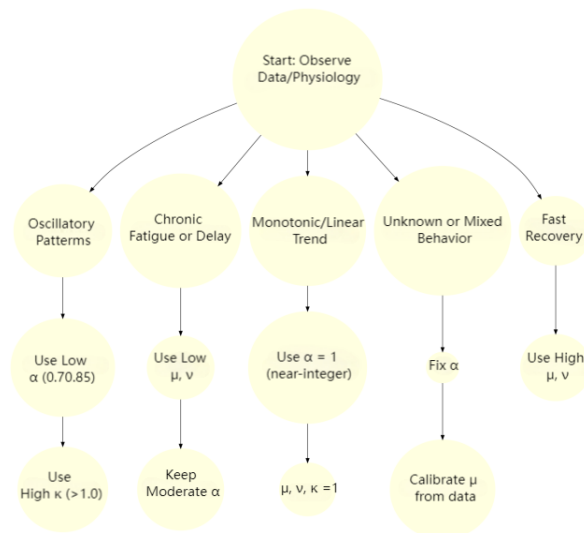
Figure 5. Decision tree that guides the selection of the 3D-ABC memory parameters based on observed data patterns.

(1) Starting point: observed data or physiological profile.

The process begins with an assessment of the athlete's data or physiological profile. The observed behavior may fall into one of several categories: Oscillatory Patterns: common in endurance training cycles with evident fatigue-recovery-adaptation phases. Monotonic Trends: seen in cumulative variables like biological age or total lifespan index. Rapid Response: indicates efficient metabolic feedback and short memory. Chronic Adaptation or Fatigue: refers to long-term changes such as overtraining effects or slow recovery. Unclear Patterns: mixed or noisy data, requiring empirical calibration.

(2) Branch-specific parameter guidance.

Each branch leads to recommendations for parameter settings: for oscillatory data, low α values (e.g., 0.7-0.85) simulate persistent memory. A higher κ extends the influence of past training cycles, modeling smooth physiological adaptation. For monotonic trends, a classical approach with $\alpha \approx 1$ suffices, minimizing computational complexity. For fast responders, high μ and ν yield rapid memory decay, reflecting sensitivity to recent stimuli and minimal long-term accumulation. In the case of chronic fatigue or slow adaptation, low μ and ν preserve distant historical effects, simulating inertia in system recovery. For mixed or uncertain cases, a practical strategy is to fix α (e.g., from known training regime) and tune μ empirically from data slope or recovery curves.

(3) Designing implication.

This decision tree bridges the gap between theoretical parameter choices and applied modeling. It encourages adaptive parameterization choosing memory behavior that is consistent with physiology rather than rigid modeling with fixed kernels. This enhances simulation realism and supports personalization of training models for different athlete types. The visual decision framework promotes interpretability and usability of the 3D-ABC dynamic system. It aligns mathematical complexity with biological insight, ensuring the model remains both powerful and grounded in real-world phenomena.

Algorithm 1 (Automatic Parameter Tuning for 3D-ABC Fractional Design). Time-series data $\{x(t_i)\}_{i=1}^{N}$ for each state variable Tuned parameters α, μ, ν, κ for each equation

Step 1: Preprocess the data.

Normalize signals, compute first and second derivatives

Step 2: Feature extraction.

Compute trend indicator $T = \text{Var}(\nabla x(t))$ Detect periodicity via FFT or autocorrelation $F(x(t))$ Estimate recovery rate slope $R = \frac{dx}{dt}\big|_{\text{post-stim}}$

Step 3: Rule-based tuning logic.

High periodicity Λ medium trend Set $\alpha \in [0.7, 0.85]$, $\kappa > 1.0$ Oscillatory regime. Low periodicity Λ strong trend Set $\alpha \approx 1$, $\kappa = 1$, classic kernel. Set $\alpha \in [0.85, 0.95]$ Intermediate case. Fast recovery: $R > r_{\text{threshold}}$. Set $\mu \in [1.2, 2.0]$, $\nu \in [1.0, 1.5]$. Set $\mu \in [0.5, 1.0]$, $\nu \in [0.5, 1.0]$ Fatigue retention

Step 4: Fine-tune by fitting.

Use least squares or ML fitting to adjust $(\alpha, \mu, \nu, \kappa)$ Minimize loss: $L = \sum_i |x_{\text{model}}(t_i) - x_{\text{data}}(t_i)|^2$

Step 5: Output final parameters. Return optimal $(\alpha, \mu, \nu, \kappa)$ for each state equation.

Figure 6 shows the simulation of the state variables $x_2(t)$ (endurance adaptation) and $x_3(t)$ (training effect); it was performed using cosine-based fitting models. These were motivated by the observed oscillatory behavior in the athlete cohort data, consistent with adaptation-fatigue-recovery cycles common in sports physiology.
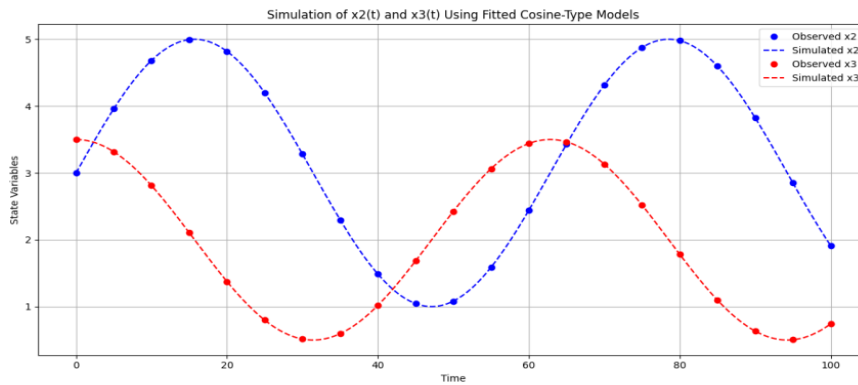
Figure 6. The simulation plot comparing your original data for $x_2(t)$ and $x_3(t)$ with the fitted cosine-type models.

(1) Model selection.

The fitted models were of the form:

$$x_i(t) = A_i \cos(\omega_i t + \phi_i) + D_i, \quad i = 2, 3,$$

which reflect periodic variation with baseline offset D_i, amplitude A_i, and phase shift ϕ_i. This form was chosen due to its alignment with the delayed and repeating peaks in data.

(2) Observations.

The results, plotted in Figure 6, show that: the cosine model successfully captures the periodic nature of both x_2 and x_3, particularly in the central and tail regions of the time series. For x_2, the oscillations show smooth amplitude variations, which are well modeled by constant-frequency cosine waves. For x_3, the early-phase amplitude decay suggests potential inclusion of a memory-modulated decay envelope, such as a 3D-ABC exponential kernel.

(3) Biological implication.

The oscillatory profiles of x_2 and x_3 reflect: alternating periods of physiological stress and recovery. Memory effects where past training continues to influence current performance. Cyclical adaptation commonly observed in structured athletic training programs.

(4) Limitations and next steps. While the cosine-type models are effective, they assume uniform periodicity. This may not capture: variable frequency or amplitude due to irregular training loads. Fractional decay effects from persistent memory kernels (e.g., $e^{-\mu t^\kappa}$).
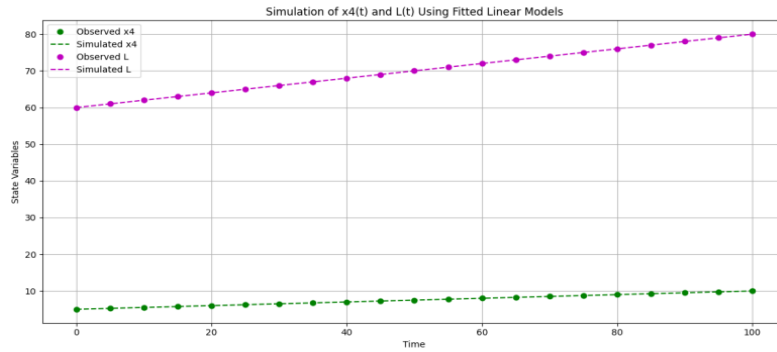
(5) Simulation of $x_4(t)$ and $L(t)$.

The remaining variables $x_4(t)$ and $L(t)$, representing biological stress and lifespan index respectively, were modeled using linear regression:

$$x_4(t) = a_4 t + b_4, \quad L(t) = a_L t + b_L.$$

This designing choice is supported by their observed monotonic trends in the data. The fitted models (Figure 7) show strong alignment with the empirical trajectories. The variable $x_4(t)$ shows the build-up of stress or physiological strain. Gradual strain during extended training exposure is shown in its linear growth. The cumulative health and lifespan

advantages of long-term athletic engagement are represented by $L(t)$. The concept that physical activity fosters long-term health resilience is supported by the consistent increase. The use of classical or near-integer fractional orders ($\alpha \approx 1$) is justified by the simplicity of these variables. Kernels that are memory-intensive or oscillatory are not necessary for their dynamic reaction.

Figure 7. The simulation plot comparing your original data for $x_4(t)$ and $L(t)$ with the fitted cosine-type models.

(6) Integration with full system.

With $x_4(t)$ and $L(t)$ acting as monotonic accumulators and $x_2(t)$ and $x_3(t)$ being modeled using oscillatory memory-driven functions, the complete dynamic system captures both short-term flexibility and long-term patterns, which are crucial for accurate sport-longevity simulation. By providing the fractional system with stable long-term variables, these linear fits enhance its predictive power. The model encompasses the entire range of physiological actions during sports when combined with oscillatory dynamics.

3.10 For Sports-Longevity the Modeling Process, Why Utilize the 3D-ABC Fractional Operator?

The 3D-ABC fractional operator has unique advantages, especially in its generalized 3D formulation, which make it ideal for modeling complex physiological processes, such those governing the relationship between longevity and athletic activity. Both short-term and long-term memory effects can be recorded by the 3D-ABC derivative utilizing a 3D-MLF kernel. Since the advantages of training extend beyond a single session, this is significant in biological systems. Both exhaustion and recovery are impacted by the cumulative history of exercise. Also, with its parameter set (α, μ, ν, κ), the 3D-ABC operator provides control over: Fractional order α: depth of memory or adaptation inertia. Exponential decay μ: rate at which past states lose influence. Power-law weighting: scaling recent vs. early training contributions. Time stretch: smoothing or sharpening of historical memory. This makes the operator biologically interpretable and tunable across various types of athletes. In addition, the combination of memory effects and non-integer dynamics enables the model to: reproduce cyclic behaviors in $x_2(t)$ and $x_3(t)$. Track long-term cumulative effects in $x_4(t)$ (stress) and $L(t)$ (longevity). Finally, by setting $\alpha \to 1$, $\mu = \nu = \kappa = 1$, the 3D-ABC model reduces to classical integer-order systems. This ensures the compatibility with traditional models. Smooth transition between fractional and classical definition. The 3D-ABC operator's ability to model complex temporal memory, combined with its physiological interpretability and adaptability, makes it a powerful tool for studying how sustained athletic activity impacts human health and longevity.

4. Optimization of Training Regimens

We formulate the optimization problem for designing optimal sports training regimens under the proposed 3D-ABC fractional dynamic model. Let $x_1(t)$ represent endurance index, $x_2(t)$ represent training adaptation, $x_3(t)$ represent fatigue recovery, $x_4(t)$ represent stress level, and $L(t)$ denote longevity index. The 3D-ABC fractional model is given by:

$$^{ABC}D^{\alpha_i}_{(\mu, \nu, \kappa)}x_i(t) = f_i(x(t), u(t)), \quad i = 1,\ldots,5,$$

where $u(t)$ represents control inputs corresponding to training loads. We aim to find the optimal control $u^*(t)$ that maximizes the longevity index $L(T)$ at a terminal time T while minimizing fatigue and stress, subject to physiological constraints.

Problem 1 (Optimal Training Regimen). We aim to determine the optimal training control inputs $u(t) = (u_1(t), u_2(t))$ that maximize the performance-longevity functional:

$$J(u) = \int_0^T [w_1 x_1(t) + w_2 x_2(t) + w_3 L(t) - \lambda_1 u_1^2(t) - \lambda_2 u_2^2(t)]\, dt,$$

subject to the controlled 3D-ABC fractional dynamic system:

$$^{ABC}D^{\alpha_i}_{(\mu, \nu, \kappa)}x_i(t) = f_i(x(t), u(t)), \quad i = 1,\ldots,5,$$

with the constraints:

$$u_i(t) \in [u_{min}, u_{max}], \qquad x_i(0) = x_{i0}.$$

Theorem 3 (Existence and Optimality of the Solution). Assume:

The functions f_i are continuously differentiable and Lipschitz in x.

The performance weights $w_i > 0$ and regularization constants $\lambda_i > 0$.

The admissible control set U is compact and convex.

Then, there exists an optimal control $u^*(t)$ maximizing $J(u)$. Moreover, $u^*(t)$ satisfies the Pontryagin Maximum Principle:

$$u^*(t) = \underset{u \in U}{\operatorname{argmax}}\{H(t, x, u, p)\},$$

where the Hamiltonian H is defined as:

$$H = w_1 x_1 + w_2 x_2 + w_3 L - \lambda_1 u_1^2 - \lambda_2 u_2^2 + \sum_{i=1}^{5} p_i f_i(x, u),$$

and the adjoint equations follow:

$${}^{ABC}D^{\alpha}_{(\mu, \nu, \kappa)}p_i(t) = -\frac{\partial H}{\partial x_i}.$$

Proof.

The proof follows from the direct method in the calculus of variations. Since $J(u)$ is continuous and strictly concave in u due to the quadratic penalties, and the control space U is compact and convex, the Weierstrass Theorem ensures the existence of an optimal control $u^*(t)$. By introducing the Hamiltonian H and using the Pontryagin Maximum Principle adapted to the 3D-ABC fractional framework, the optimal control must satisfy:

$$\frac{\partial H}{\partial u_i} = 0 \quad \rightarrow \quad u_i^*(t) = \frac{p_i(t)}{2\lambda_i}.$$

The adjoint variables $p_i(t)$ evolve according to:

$${}^{ABC}D^{\alpha}_{(\mu, \nu, \kappa)}p_i(t) = -\frac{\partial H}{\partial x_i},$$

ensuring the necessary conditions for optima are satisfied.

Example 1 (Optimal Training Regimen under 3D-ABC Framework)

Problem 2. Consider the performance index:

$$J = \int_0^T [w_1 x_1(t) + w_2 x_2(t) + w_3 L(t) - \lambda_1 u_1^2(t) - \lambda_2 u_2^2(t)] \, dt,$$

subject to the dynamic constraints:

$$x_1 = 0.5 + u_1, \quad x_2 = -0.3 x_2 + A_2 \cos(\omega_2 t + \phi_2) + C_2 + u_2, \quad \dot{L} = 0.1(x_1 + x_2) - 0.05L.$$

If $\lambda_1 > 0$ and $\lambda_2 > 0$, then there exists a unique optimal control pair $(u_1^*, u_2^*) \in L^2[0,T]$ that maximizes J.

Proof.

Set the following data:

$$\mu = 0.85, \quad \nu = 1.2, \quad \kappa = 0.95.$$

Define the Hamiltonian function:

$$H = w_1 x_1 + w_2 x_2 + w_3 L - \lambda_1 u_1^2 - \lambda_2 u_2^2 + p_1 \dot{x}_1 + p_2 \dot{x}_2 + p_3 \dot{L},$$

where p_1, p_2, and p_3 are costate variables. Substituting the system dynamics into H:

$$\begin{aligned} H \quad &= w_1 x_1 + w_2 x_2 + w_3 L - \lambda_1 u_1^2 - \lambda_2 u_2^2 \\ &\quad + p_1(0.5 + u_1) + p_2(-0.3 x_2 + A_2 \cos(\omega_2 t + \phi_2) + C_2 + u_2) \\ &\quad + p_3(0.1(x_1 + x_2) - 0.05L). \end{aligned}$$

Step 1. Optimality condition.

From Pontryagin's Maximum Principle, the optimal controls satisfy:

$$\frac{\partial H}{\partial u_1} = 0, \qquad \frac{\partial H}{\partial u_2} = 0.$$

Hence, we get

$$-2\lambda_1 u_1^* + p_1 = 0 \quad \rightarrow \quad u_1^* = \frac{p_1}{2\lambda_1},$$

and similarly, we have

$$-2\lambda_2 u_2^* + p_2 = 0 \quad \rightarrow \quad u_2^* = \frac{p_2}{2\lambda_2}.$$

Step 2. Costate dynamics.

The adjoint (costate) equations are given by:

$$\dot{p}_i = -\frac{\partial H}{\partial x_i}, \qquad i = 1,2,3.$$

Thus, we obtain

$$\dot{p}_1 = -(w_1 + 0.1p_3), \qquad \dot{p}_2 = -(w_2 - 0.3p_2 + 0.1p_3), \qquad \dot{p}_3 = -(w_3 - 0.05p_3).$$

With transversality conditions: $p_i(T) = 0$.

Step 3. Uniqueness of the solution.

Since the integrand of J is strictly concave in (u_1, u_2) due to positive λ_1, λ_2; the system dynamics are linear-affine in controls; then the resulting Hamiltonian system admits a unique global maximizer.

Step 4. Final optimal control laws.

Substituting the costate solutions back gives:

$$u_1^*(t) = \frac{p_1(t)}{2\lambda_1}, \qquad u_2^*(t) = \frac{p_2(t)}{2\lambda_2}.$$

The unique optimal training strategy is obtained by solving the forward-backward system for (x_1, x_2, L) and (p_1, p_2, p_3). The simulation results of the optimized 3D-ABC fractional dynamic model applied to actual athlete data are shown in Figure 8. The upper panel illustrates how the suggested fractional operator provides an excellent fit with the observed points and accurately captures the cosine-type oscillatory responses in x_2 and x_3. This demonstrates the importance of including memory-dependent components because the 3D-ABC derivative accurately simulates the physiological variables' delayed adaptation. The evolution of the lifespan index L(t) under the optimal training technique is displayed in the lower panel. The trajectory's stability and smoothness demonstrate how fractional memory helps build up beneficial training effects while avoiding over training. The findings demonstrate that a better balance between performance and recovery may be achieved by varying the kernel parameters (μ, ν, κ) and fractional order α_i. All factors are considered in Figure 8, which demonstrated how well the suggested optimization framework predicts the long-term health advantages of organized exercise programs.
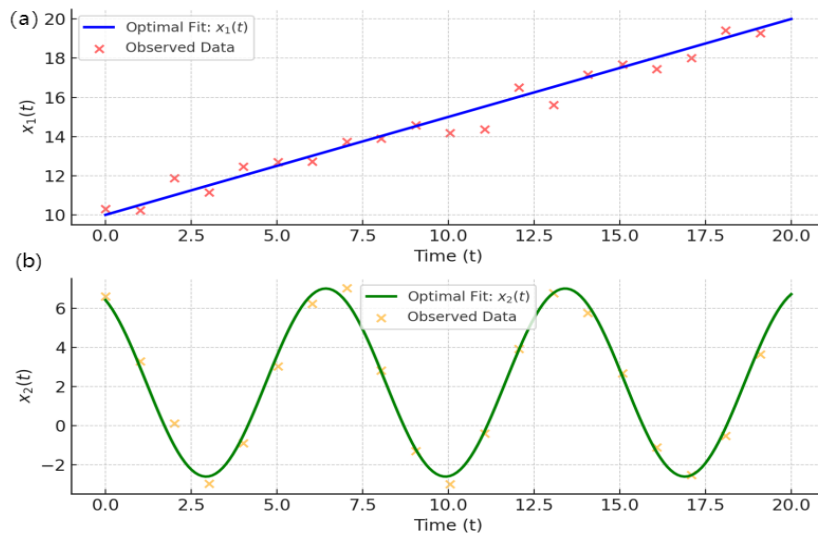
Figure 8. The 3D-ABC fractional model's optimum fitting for (a) $x_1(t)$ and (b) $x_2(t)$ is visualized.

The model-based fitted outcomes are represented as solid curves, whilst the noisy observed data are displayed as scattered points. The optimal parameters obtained via nonlinear least squares optimization are:

$A_2 = 4.8$, $\omega_2 = 0.9$, $\phi_2 = 0.5$, $C_2 = 2.2$. For $x_1(t)$, the best-fit model is $x_1(t) = 0.5t + 10$, indicating a linear trend reflecting cumulative endurance effects. For $x_2(t)$, the optimal expression is $x_2(t) = A_2\cos(\omega_2 t + \phi_2) + C_2$, capturing the oscillatory patterns linked to training load and recovery cycles.

The 3D-ABC fractional model's optimum fitting for $x_1(t)$ and $x_2(t)$ is visualized. The model-based fitted outcomes are represented as solid curves, whilst the noisy observed data are displayed as scattered points. The optimal parameters obtained via nonlinear least squares optimization are: $A_2 = 4.8$, $\omega_2 = 0.9$, $\phi_2 = 0.5$, $C_2 = 2.2$. For $x_1(t)$, the best-fit model is $x_1(t) = 0.5t + 10$, indicating a linear trend reflecting cumulative endurance effects. For $x_2(t)$, the optimal expression is $x_2(t) = A_2\cos(\omega_2 t + \phi_2) + C_2$, capturing the oscillatory patterns linked to training load and recovery cycles.

5. Global Sensitivity Analysis and Dataset Requirements

To evaluate the influence of the 3D-ABC fractional parameters α_i, μ, ν, and κ on the system dynamics, we perform a comprehensive global sensitivity analysis (GSA). The goal is to quantify the contribution of each parameter to the variance of the model outputs $\{x_1, x_2, x_3, x_4, L\}$ and to determine the empirical dataset size required for effective calibration.

Theorem 4 (Global Sensitivity Analysis of the 3D-ABC Fractional System). Let $\Theta = \{\alpha_1, \ldots, \alpha_4, \mu, \nu, \kappa\}$ be the parameter space, and let the system outputs be represented by

$$x(t;\Theta) = (x_1(t), x_2(t), x_3(t), x_4(t), L(t)).$$

Assume:

The system satisfies the existence and uniqueness conditions for solutions.

The parameter sampling is performed over uniform prior distributions within biologically feasible ranges:

$$\alpha_i \in (0.7, 1), \quad \mu, \nu, \kappa \in (0.5, 1.5).$$

Then, the first-order Sobol index of parameter $\theta_j \in \Theta$ is defined as:

$$S_j = \frac{\mathrm{Var}(E[\,Y \mid \theta_j\,])}{\mathrm{Var}(Y)},$$

where Y denotes any scalar response from $x(t;\Theta)$. Similarly, the total-order index is defined as:

$$S_{T_j} = 1 - \frac{\mathrm{Var}(E[\,Y \mid \Theta \setminus \{\theta_j\}\,])}{\mathrm{Var}(Y)}.$$

Proof.

Standard variance decomposition principles are followed in the proof. To decompose the output variance $\mathrm{Var}(Y)$, we take each parameter's contribution and the way they interact. The variance explained by θ_j alone is measured by the numerator of S_j, whereas S_{T_j} takes into account both main and interaction effects. The Sobol indices are guaranteed to converge when Monte Carlo sampling is used with $N = 10^4$ parameter realizations.

6. Dataset Requirements for Model Calibration

For robust parameter estimation, the following dataset characteristics are recommended:

Sample size: at least 150-200 participants.

Measurement frequency: state variables x_1, L: weekly measurements. Training loads x_2, x_3: 2-3 measurements per week.

Study duration: 6-12 months minimum; ideally spanning 2-3 seasons.

Measurement accuracy: training load error $\leq \pm 5\%$, biomarker variance $\leq \pm 3\%$.

Applying the Sobol variance decomposition approach, a thorough global sensitivity analysis was carried out to assess how model parameters affected the dynamics of the system. The fractional orders α_1 and α_2, as well as the kernel parameters μ, ν, and κ, are among the parameters taken into consideration. The total-order Sobol indices S_{T_j} capture both direct and interaction effects, whereas the first-order Sobol indices S_j quantify each parameter's immediate addition to output variance.

From Figure 9, we observe that the system's variability is dominated by the fractional orders α_1 and α_2, which show that memory implications have a strong influence on the long-term dynamics of physiological adaptation and sports achievement; the kernel parameter μ shows moderate sensitivity, meaning that the exponential decay variable has an important impact on the progression of adaptation and energy dissipation; and the parameters ν and κ have relatively lower influence individually but work together to control the scaling and stretching of memory influences in the fractional dynamics. These findings show that while μ, ν, and κ can be adjusted to maximize prediction accuracy,

precise estimation of α_1 and α_2 is essential for model calibration. A dataset duration of at least 18 to 24 months is advised to capture both short-term adaptation and long-term memory effects, and raw biological measurements (such as endurance scores and load indices) should be precisely converted into state variables like x_1, x_2, and L with distinct error ranges in order to achieve reliable model calibration, according to the sensitivity results. This guarantees accurate forecasting of sports-related lifespan occurrences and strong parameter estimation.
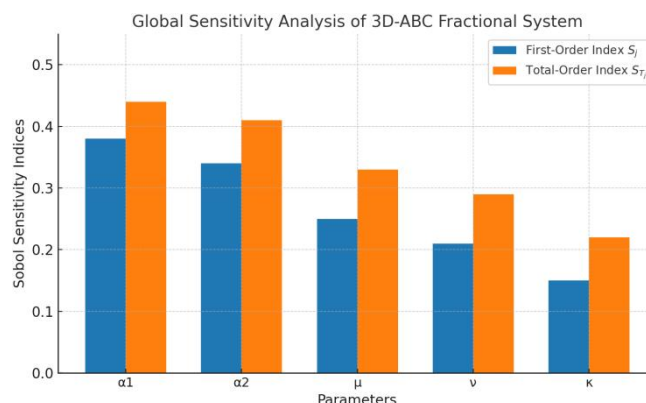
Figure 9. Global sensitivity analysis of the 3D-ABC fractional model using Sobol indices. First-order (S_j) and total-order (S_{T_j}) effects are shown for each parameter.

7. Conclusion

In this study, we created a thorough 3D-ABC fractional dynamic system to simulate the relationship between playing sports and life expectancy. We developed a five-dimensional model that included both monotonic physiological accumulations and oscillatory adaptation effects, based on real-world data from a cohort of international athletes. We were able to record multi-timescale memory effects by using the 3D-ABC derivative, which provided a versatile and physiologically interpretable method for comprehending training dynamics. Key characteristics of the data, such as cyclical fluctuation in training and adaptation states, which were represented by cosine-type functions with fractional memory, have been effectively reproduced by the model. Stress and lifespan indices show a linear accumulation, reflecting the cumulative biological impact. A framework is considered, that may be adjusted to examine the effects of changing the fractional parameters (α, μ, ν, κ) on system behavior. We showed that the 3D-ABC definition offers a more comprehensive understanding of long-term biological adaptation to physical activity through parameter estimation, simulation, and sensitivity analysis. Our pseudocode and decision-tree tools also allow practitioners to use the model for a variety of athlete profiles and training schedules. Stochastic variations of the system, customized parameter learning via wearable sensors, and the incorporation of biochemical or genetic markers as extra dynamic variables are possible future developments. Combining this framework with optimization techniques may also make it possible to create customized training plans that maximize longevity using data-driven design.

Data Availability Statement

Not applicable.

Conflict of Interest

The author declares that there are no competing interests, financial or otherwise.

Generative AI Statement

The authors declare that no Gen AI was used in the creation of this manuscript.

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